Multinational Production, Risk Sharing,
and Home Equity Bias

Technical Appendix

Not for Publication

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1 Model Details

This Appendix shows derivations for Section 2.

1.1 Derivation of price indices, demand for goods, and real exchange rate

First, we derive the price index in the home country, P_t. It consists of the price index of goods produced by home firms in the home country, P_{Ht}, and price index of goods produced by foreign firms in the home country, P_{Ft}. C_t is the home consumer's consumption basket

Now, we derive the price index of goods produced by home firms in the home country P_{Ht}.
In this derivation, the home consumer's consumption basket, C_{Ht}, consists of goods produced
by the home firms z where we integrate from 0 to a because there are a home firms:

$\min p_t(z) c_t(z)$ subject to $C_{Ht} = 1$ where $C_{Ht} = [(\frac{1}{a})^{\frac{1}{\theta}} \int_0^a c_t(z)^{\frac{\theta-1}{\theta}} dz]^{\frac{\theta}{\theta-1}}$

$L = p_t(z) c_t(z) - P_{Ht} [[(\frac{1}{a})^{\frac{1}{\theta}} \int_0^a c_t(z)^{\frac{\theta-1}{\theta}} dz]^{\frac{\theta}{\theta-1}} - 1]$

$\frac{\partial L}{\partial c_t(z)} = p_t(z) - P_{Ht} \frac{\theta}{\theta-1} [(\frac{1}{a})^{\frac{1}{\theta}} \int_0^a c_t(z)^{\frac{\theta-1}{\theta}} dz]^{\frac{\theta}{\theta-1}-1} (\frac{1}{a})^{\frac{1}{\theta}} \frac{\theta-1}{\theta} c_t(z)^{\frac{\theta-1}{\theta}-1} = 0$

$p_t(z) = P_{Ht} [(\frac{1}{a})^{\frac{1}{\theta}} \int_0^a c_t(z)^{\frac{\theta-1}{\theta}} dz]^{\frac{1}{\theta-1}} (\frac{1}{a})^{\frac{1}{\theta}} c_t(z)^{-\frac{1}{\theta}}$

$c_t(z)^{\frac{1}{\theta}} = \frac{p_t(z)}{P_{Ht}} a^{\frac{1}{\theta}}$

$c_t(z) = \frac{1}{a} (\frac{p_t(z)}{P_{Ht}})^{-\theta}$ [2]

Substitute this expression into $C_{Ht} = 1$:

$[(\frac{1}{a})^{\frac{1}{\theta}} \int_0^a (\frac{P_{Ht}}{p_t(z)})^{\theta-1} (\frac{1}{a})^{\frac{\theta-1}{\theta}} dz]^{\frac{\theta}{\theta-1}} = 1$

$[(\frac{1}{a})^{\frac{1}{\theta}} (\frac{1}{a})^{\frac{\theta-1}{\theta}} \int_0^a (\frac{P_{Ht}}{p_t(z)})^{\theta-1} dz]^{\frac{\theta}{\theta-1}} = 1$

$P_{Ht} [\frac{1}{a} \int_0^a (\frac{1}{p_t(z)})^{\theta-1} dz]^{\frac{\theta}{\theta-1}} = 1$

$[\frac{1}{a} \int_0^a p_t(z)^{1-\theta} dz]^{\frac{\theta}{\theta-1}} = P_{Ht}$

$[\frac{1}{a} \int_0^a p_t(z)^{1-\theta} dz]^{-\frac{1}{\theta-1}} = P_{Ht}$

$[\frac{1}{a} \int_0^a p_t(z)^{1-\theta} dz]^{\frac{1}{1-\theta}} = P_{Ht}$, which is the price index of goods produced by home firms (de-
noted by z) in the home country.

We can then write the demand for home firm z output by the representative household in
the home country based on the above as:

$c_t(z) = \frac{1}{a} (\frac{p_t(z)}{P_{Ht}})^{-\theta} C_{Ht} = \frac{1}{a} (\frac{p_t(z)}{P_{Ht}})^{-\theta} (\frac{P_{Ht}}{P_t})^{-\eta} a C_t = (\frac{p_t(z)}{P_{Ht}})^{-\theta} (\frac{P_{Ht}}{P_t})^{-\eta} C_t$ [3].

Since there are a home households, the demand for home firm z output by all households in
the home country is: $(\frac{p_t(z)}{P_{Ht}})^{-\theta} (\frac{P_{Ht}}{P_t})^{-\eta} a C_t$.

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[2] Note that this expression should be completely written as $c_t(z) = \frac{1}{a} (\frac{P_{Ht}}{p_t(z)})^{\theta} C_{Ht}$ but we drop C_{Ht} because
we imposed $C_{Ht} = 1$.

[3] Note that in this expression we should write $(C_t + G_t)$ to reflect the total demand made by the home
country that comes from home consumers as well as home government. However, for the purpose of this
derivation, we can omit G_t.

The demand for home firm z output by all households and government in the home country is: $\left(\frac{p_t(z)}{P_{Ht}}\right)^{-\theta}\left(\frac{P_{Ht}}{P_t}\right)^{-\phi}(aC_t + aG_t)$ assuming that the government spends G_t per capita. Notice: $a(C_t + G_t)$ is Y_t^d, i.e, demand for consumption basket in the home country. Note that in contrast to Ghironi, Lee, and Rebucci (2015) (GLR), we do not have Y_t^W. Note: The total per capita demand for consumption basket in the home country is $Y_t^d = C_t + G_t$

The price index of goods produced by foreign firms in the home country can be derived by following the same steps. In this derivation, the home consumer's consumption basket, C_{Ft}, consists of goods produced by the foreign firms z where we integrate from a to $1-a$ because there are $1-a$ foreign firms:

$[\frac{1}{1-a}\int_a^1 p_t(z)^{1-\theta} dz]^{\frac{1}{1-\theta}} = P_{Ft}$ using consumption of goods produced by foreign firms in the home country, $C_{Ft} = [(\frac{1}{1-a})^{\frac{1}{\theta}}\int_a^1 c_t(z)^{\frac{\theta-1}{\theta}} dz]^{\frac{\theta}{\theta-1}}$

The derivation of the price index of goods produced in the foreign country (consisting of a price index of goods produced by home firms in the foreign country, P^*_{Ht}, and a price index of goods produced by foreign firms in the foreign country, P^*_{Ft}), P^*_t, yields:

$P^*_t = [aP_{Ht}^{*1-\phi} + (1-a)P_{Ft}^{*1-\phi}]^{\frac{1}{1-\phi}}$

Note that the expressions for P_{Ht}, P_{Ft}, P^*_{Ht} and P^*_{Ft} (and, hence, P_t and P^*_t) are identical to GLR. However, since purchasing power parity does not hold in our model, we have to take the real exchange rate, Q_t, into account.

$Q_t \equiv \frac{\varepsilon_t P^*_t}{P_t}$ where ε_t is the nominal exchange rate, and $\varepsilon_t P^*_t = [a(\varepsilon_t P^*_{Ht})^{1-\phi} + (1-a)(\varepsilon_t P^*_{Ft})^{1-\phi}]^{\frac{1}{1-\phi}}$.
Then: $Q_t = [\frac{a(\varepsilon_t P^*_{Ht})^{1-\phi} + (1-a)(\varepsilon_t P^*_{Ft})^{1-\phi}}{aP_{Ht}^{1-\phi} + (1-a)P_{Ft}^{1-\phi}}]^{\frac{1}{1-\phi}}$

1.2 Household optimization

Start with the home household budget constraint in nominal terms in home currency:

$(V_t + D_t + \varepsilon_t D^*_t)x_t + (\varepsilon_t V^*_t + D^*_t + \varepsilon_t D^*_t)x^*_t + W_t L_t = V_t x_{t+1} + \varepsilon_t V^*_t x^*_{t+1} + P_t C_t + P_t G_t,$

where x_t denotes shares of the home firm, x^*_t denotes shares of the foreign firm, V_t is the price of the home firm's shares, V^*_t is the price of the foreign firm's shares, D_t is the dividend

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With respect to x_{t+1} :

$$\frac{@}{@x_{t+1}} = _t(\ v_t) + \ E_t f_{t+1}(v_{t+1} + d_{t+1} + d_{t+1})g = 0$$
$$C_t^{-1} v_t = \ E_t f C_{t+1}^{-1}(v_{t+1} + d_{t+1} + d_{t+1})g$$

home rm's dividends coming from

With respect to x_{t+1} :

$$\frac{@}{@x_{t+1}} = _t(\ v_t) + \ E_t f_{t+1}(v_{t+1} + d_{t+1} + d_{t+1})g = 0$$

subject to:

$Y_t^s(z) = Y_t^d(z)$, which says that output supplied by the home firm in the home country has

to equal this firm's output demanded in the home country,

and

$Y_t^s(z) = Y_t^d(z)$, which says that output supplied by the home firm in the foreign country

has to equal this firm's output demanded in the foreign country.

To derive the optimal demand for labor by home firm, z, in the home country, we use $Y_t^s(z) = Y_t^d(z)$. $Y_t^s(z)$ comes from the production function, i.e., $Y_t^s(z) = Z_t L_t(z)$. $Y_t^d(z)$ comes from the demand for home firm's z good that was derived in Section 1.1: $Y_t^d(z) = (\frac{P_t(z)}{P_{Ht}})^{-\eta} (\frac{P_{Ht}}{P_t})^{-\theta} (aC_t + aG_t)$ (which is $Y_t^d(z) = (\frac{P_t(z)}{P_{Ht}})^{-\eta} (\frac{P_{Ht}}{P_t})^{-\theta} Y_t^d$ because $(aC_t + aG_t)$ rep-

resents the demand by all home firms

aC.83Td(Z)0701 Tf 15.378 -1.794 Td [(t)]TJ/F18 11.69 -1.339 Td [(H)-75(t)]TJ ET q 1 0 0 1 359.842 50

zzJTd05 cm []0 d 0 J 0.478 w 0 0 0468.473 0 l S Q BT /F35 7.9701 Tf 320.268 496.794 Td [(

subject to:

$$Y_t^s($$

$p_t(z) = \dfrac{W_t}{1 \ Z_t \ Z_t^{1}}$, which is the price charged by the home firm in the foreign country.

For the foreign firm, z, the problem becomes:

$$\text{Max } p_t(z\,)Z_t^{1} \ Z_t \ \left(\frac{p_t(z\,)}{P_{Ft}}\right) \ ^! \ \left(\frac{P_{Ft}}{P_t}\right) \ ^! \ \frac{a(C_t+G_t)}{Z_t^{1} \ Z_t} + \ ''_t p_t(z\,)Z_t\left(\frac{p_t(z\,)}{P_{Ft}}\right) \ \left(\frac{P_{Ft}}{P_t}\right) \ ^! \ \frac{(1 \ a)(C_t+G_t)}{Z_t}$$

$$W_t\left(\frac{p_t(z\,)}{P_{Ft}}\right) \ \left(\frac{P_{Ft}}{P_t}\right) \ ^! \ \frac{a(C_t+G_t)}{Z_t^{1} \ Z_t} \ \ ''_t W_t\left(\frac{p_t(z\,)}{P_{Ft}}\right) \ \left(\frac{P_{Ft}}{P_t}\right) \ ^! \ \frac{(1 \ a)(C_t+G_t)}{Z_t}$$

Take the derivative with respect to $p_t(z\,)$:

$(1 \quad) = \dfrac{W_t}{Z_t^{1} \ Z_t \ p_t(z\,)}$

$p_t(z\,) = \dfrac{W_t}{1 \ Z_t^{1} \ Z_t}$, which is the price charged by the foreign firm in the home country.

Take the derivative with respect to $p_t(z\,)$:

$(1 \quad) = \dfrac{W_t}{Z_t \ p_t(z\,)}$

$p_t(z\,) = \dfrac{W_t}{1 \ Z_t}$, which is the price charged by the foreign firm in the foreign country.

In equilibrium, $p_t(z) = P_{Ht}$, which says that price charged by home firm z in home country equals the price index for goods produced by home firms. Similarly, $p_t(z) = P_{Ht}$ for price charged by home firms in the foreign country, $p_t(z\,) = P_{Ft}$ for price charged by foreign firms in the home country, and $p_t(z\,) = P_{Ft}$ for price charged by foreign firms in the foreign country.

Therefore:

$P_{Ht} = \dfrac{W_t}{1 \ Z_t}$ for price index of goods produced by home firms in the home country,

$P_{Ht} = \dfrac{W_t}{1 \ Z_t \ Z_t^{1}}$ for price index of goods produced by home firms in the foreign country,

$P_{Ft} = \dfrac{W_t}{1 \ Z_t^{1} \ Z_t}$ for price index of goods produced by foreign firms in the home country,

and

$P_{Ft} = \dfrac{W_t}{1 \ Z_t}$ for price index of goods produced by foreign firms in the foreign country.

Then, we can write expressions for relative prices:

$RP_t = \dfrac{p_t(z)}{P_t} = \dfrac{P_{Ht}}{P_t} = \dfrac{W_t}{1 \ Z_t}$ for price charged by a home firm in the home country relative to the home country's price level in units of the home country consumption,

$RP_t = \dfrac{p_t(z)}{P_t} = \dfrac{P_{Ht}}{P_t} = \dfrac{W_t}{1 \ Z_t \ Z_t^{1}}$ for price charged by a home firm in the foreign country

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relative to the foreign country's p[...] [u]nits of the for[eign] [...] [consum]ption,

$$RP_t = \frac{p_t(z)}{P_t} = \frac{P_{Ft}}{P_t} = \frac{1}{1}\frac{w_t}{Z_t^1 \, Z_t}$$ [ch]arged by a foreign [fi]rm [...] [cou]ntry

relative to the home country's pr[...] [un]its of the home count[ry...]

$$RP_t = \frac{p_t(z)}{P_t} = \frac{P_{Ft}}{P_t} = \frac{1}{1}\frac{w_t}{Z_t}$$ fo[r...] [produc]ed by a foreign [fi]rm [...]

relative to the foreign country's p[...] [u]nits of the foreign co[untry...] [fir]m[...]

Note that the small case letter, w [...] [d]enote real wage as o[...] [t]he la[...]

letter W that denotes nominal w[...]

The optimal labor demands can [...] [w]i[th rela]tive prices as [...]

Optimal demand for labor by a h[ome ...] [th]e country be[...]

$$L_t(z) = RP_t^{1} \frac{a(C_t + G_t)}{Z_t}$$ $z) = RP$ [...]

$$(1\ a)L_t\ (z\ ) = (1\ a)RP\ _t^{!}\ \frac{a(C_t+G_t)}{Z_t^1\ Z_t}$$

Per capita labor demand by all foreign rms in home country is:

$$\tfrac{1\ a}{a}L_t\ (z\ ) =\ \tfrac{1\ a}{a}RP\ _t^{!}\ \frac{a(C_t+G_t)}{Z_t^1\ Z_t}$$

where we again divide by a because there are a households in the home country.

There are a home rms in the foreign country, so the optimal demand for labor by all home rms in the foreign country is:

$$aL_t(z) =\ aRP_t\ ^{!}\ \frac{(1\ a)(C_t+G_t)}{Z_t\ Z_t^1}$$

Per capita labor demand by all home rms in foreign country is:

$$\tfrac{a}{1\ a}L_t\ (z) =\ \tfrac{a}{1\ a}RP_t\ ^{!}\ \frac{(1\ a)(C_t+G_t)}{Z_t\ Z_t^1}$$

where we divide by (1 a) because there are (1 a) households in the home country.

There are (1 a) foreign rms in the foreign country, so the optimal demand for labor by all foreign rms in the foreign country is:

$$(1\ a)L\ _t(z\ ) = (1\ a)RP\ _t\ ^{!}\ \frac{(1\ a)(C_t+G_t)}{Z_t}$$

Total per capita labor demand by all foreign rms in the foreign country is:

$$\tfrac{1\ a}{1\ a}L\ _t(z\ ) =\ \tfrac{1\ a}{1\ a}RP\ _t\ ^{!}\ \frac{(1\ a)(C_t+G_t)}{Z_t}$$

where we again divide by (1 a) because there are (1 a) households in the home country.

1.4   Net foreign assets (NFA) law of motion

Start with the home household budget constraint in units of the home country's consumption basket from Section 1.2:

$$(v_t + d_t + d_t)x_t + (\ v_t + d_t + d_t)x_t + w_tL_t = v_t x_{t+1} + v_t\ X_{t+1} + C_t + G_t$$

Then:

$$(v_t + d_t + d_t^*)x_t + (v_t^* + d_t^* + d_t^*)x_t^* + w_t L_t = v_t x_{t+1} + nfa_{t+1} + \frac{1-a}{a}v_t^* x_{t+1}^* + C_t + G_t$$

where net foreign assets, nfa_{t+1}, is defined as $nfa_{t+1} \equiv v_t x_{t+1} - \frac{1-a}{a}v_t^* x_{t+1}^*$, i.e., the value of home holdings of foreign shares minus the value of foreign holdings of home shares adjusted for population sizes of home and foreign countries, i.e., a and $1-a$, respectively, as in GLR. We defined return on holding home equity as $R_t \equiv \frac{v_t + d_t + d_t^*}{v_{t-1}}$ and return on holding foreign equity as $R_t^* \equiv \frac{v_t^* + d_t^* + d_t^*}{v_{t-1}^*}$ in Section 1.2, so:

$$v_t x_{t+1} + nfa_{t+1} + \frac{1-a}{a}v_t^* x_{t+1}^* + C_t + G_t = \frac{(v_t + d_t + d_t^*)v_{t-1}}{v_{t-1}}x_t + \frac{(v_t^* + d_t^* + d_t^*)v_{t-1}^*}{v_{t-1}^*}x_t^* + w_t L_t$$

$$v_t x_{t+1} + nfa_{t+1} + \frac{1-a}{a}v_t^* x_{t+1}^* + C_t + G_t = R_t v_{t-1}x_t + R_t^* v_{t-1}^* x_t^* + w_t L_t$$

$$nfa_{t+1} = -v_t x_{t+1} - \frac{1-a}{a}v_t^* x_{t+1}^* + R_t v_{t-1}x_t + R_t^* v_{t-1}^* x_t^* + w_t L_t - C_t - G_t$$

$$nfa_{t+1} = -v_t(x_{t+1} + \frac{1-a}{a}x_{t+1}^*) + R_t v_{t-1}x_t + R_t^* v_{t-1}^* x_t^* + w_t L_t - C_t - G_t$$

$$nfa_{t+1} = -v_t + R_t v_{t-1}x_t + R_t^* v_{t-1}^* x_t^* + w_t L_t - C_t - G_t$$

where market clearing condition $ax_{t+1} + (1-a)x_{t+1}^* = a$ was used to obtain $x_{t+1} = 1 - \frac{1-a}{a}x_{t+1}^*$ as in GLR.

$$nfa_{t+1} = -v_t + R_t^* v_{t-1}^* x_t^* + R_t v_{t-1}(1 - \frac{1-a}{a}x_t^*) + w_t L_t - C_t - G_t \text{ where we used } x_t = 1 - x_t^* \frac{1-a}{a}.$$

$$nfa_{t+1} = -v_t + R_t^* v_{t-1}^* x_t^* + R_t v_{t-1} - R_t v_{t-1}\frac{1-a}{a}x_t^* + w_t L_t - C_t - G_t$$

$$nfa_{t+1} = -v_t + R_t^* v_{t-1}^* x_t^* + v_t + d_t + d_t^* - R_t v_{t-1}\frac{1-a}{a}x_t^* + w_t L_t - C_t - G_t$$

$$nfa_{t+1} = R_t^* v_{t-1}^* x_t^* - R_t v_{t-1}\frac{1-a}{a}x_t^* + y_t - C_t - G_t$$

where $y_t \equiv d_t + d_t^* + w_t L_t$, which differs from GLR due to the additional term d_t^*. Note that we assume that the dividend of the home firm producing in the foreign country, d_t^*, is a part of the home country GDP, i.e., we assume that firms repatriate profits to their countries of origin for distribution to domestic and foreign shareholders.

$$nfa_{t+1} = R_t^* v_{t-1}^* x_t^* - R_t v_{t-1}x_t + R_t v_{t-1}x_t - R_t v_{t-1}\frac{1-a}{a}x_t^* + y_t - C_t - G_t$$

Define excess return from holding foreign equity $R_t^D = R_t^* - R_t$ and portfolio holding $\alpha_t = v_{t-1}x_t$:

$$nfa_{t+1} = R_t^D \alpha_t + R_t v_{t-1}x_t - R_t v_{t-1}\frac{1-a}{a}x_t^* + y_t - C_t - G_t$$

$$nfa_{t+1} = R_t^D \alpha_t + R_t nfa_t + y_t - C_t - G_t$$

where definition $nfa_t \equiv v_{t-1}x_t - \frac{1-a}{a}v_{t-1}^* x_t^*$ was used.

This is identical to GLR except the definitions of R_t and R_t^*, and hence R_t^D, differ as explained above. This is in units of home consumption.

Similar derivations can be done to obtain the NFA law of motion for the foreign household:

$$nfa_{t+1}^{f} = R_t^{Df} \,_t^{f} + R_t^{f} nfa_t^{f} + y_t^{f} \quad C_t^{f}$$

Derivation of home GDP, y_t, i.e., output produced by home and foreign firms in the home country:

$$y_t = RP_t Z_t L_t + RP_{*t} Z_t^{*1} Z_t L_{*t} = \frac{1}{1}\frac{w_t}{Z_t}Z_t L_t + \frac{1}{1}\frac{w_{*t}}{Z_t^{*1} Z_t}Z_t^{*1} Z_t L_{*t} = \frac{1}{1}(w_t L_t + w_{*t} L_{*t}),$$

which is in units of home country consumption.

Derivation of foreign GDP, y_{*t}, i.e., output produced by home and foreign firms in the foreign country:

$$y_{*t} = RP_{*t} Z_t Z_t^{*1} L_{*t} + RP_{*t} Z_t L_{*t} = \frac{1}{1}\frac{w_t}{Z_t Z_t^{*1}}Z_t Z_t^{*1} L_{*t} + \frac{1}{1}\frac{w_{*t}}{Z_t}Z_t L_{*t} =$$
$$= \frac{1}{1}(w_t L_{*t} + w_{*t} L_{*t}),$$

which is in units of foreign country consumption.

Expression for $\frac{y_t}{y_{*t}}$:

$$\frac{y_t}{y_{*t}} = \frac{RP_t Z_t L_t + RP_{*t} Z_t^{*1} Z_t L_{*t}}{RP_{*t} Z_t Z_t^{*1} L_{*t} + RP_{*t} Z_t L_{*t}} = \frac{\frac{1}{1}(w_t L_t + w_{*t} L_{*t})}{\frac{1}{1}(w_t L_{*t} + w_{*t} L_{*t})} = \frac{w_t(L_t + L_{*t})}{w_{*t}(L_{*t} + L_{*t})}$$

Note that we should use the real exchange rate in the relative GDP expression but it cancels because it appears on both sides of the equation: $\frac{y_t}{Q_t y_{*t}} = \frac{w_t(L_t + L_{*t})}{Q_t w_{*t}(L_{*t} + L_{*t})}$

Next, expressions for w_t, w_{*t}, $(L_t + L_{*t})$ and $(L_{*t} + L_{*t})$ are obtained. To get w_t, home labor supply and home labor demand are equated. Home labor supply was derived in Section 1.2 from home household FOCs as $L_t^s = (\frac{C_t^{1} w_t}{})'$. Home labor demand was derived above from firm FOCs in Section 1.3 as $L_t^d = RP_t^{!} \frac{a(C_t + G_t)}{}$

(c ¹

$$\frac{y_t}{y_t} = \Big[\frac{C_t+G_t}{C_t+G_t}\Big]^{\frac{1-!}{-+!}}\Big[\frac{aZ_t^{!-1}+(1-a)(Z_t^1 Z_t)^{!-1}}{a(Z_t Z_t^{-1})^{!-1}+(1-a)Z_t^{!-1}}\Big]^{\frac{1+'}{!-1}\frac{1-!}{-+!}}\Big[\frac{C_t+G_t}{C_t+G_t}\Big]^{\frac{1+'}{-+!}}\Big[\frac{aZ_t^{!-1}+(1-a)(Z_t^1 Z_t)^{!-1}}{a(Z_t Z_t^{-1})^{!-1}+(1-a)Z_t^{!-1}}\Big]^{\frac{1+'}{-+!}} =$$

$$= \Big[\frac{C_t+G_t}{C_t+G_t}\Big]^{\frac{(!-1)+(1+')}{-'+!}} + \Big[\frac{aZ_t^{!-1}+(1-a)(Z_t^1 Z_t)^{!-1}}{a(Z_t Z_t^{-1})^{!-1}+(1-a)Z_t^{!-1}}\Big]^{\frac{(1+')(1-!)+(1+')(!-1)}{(!-1)('+!)}} =$$

$$= \frac{C_t+G_t}{C_t+G_t}$$

1.6 More on real exchange rate, Q_t

From Section 1.1: $Q_t = \Big[\frac{a("_t P_{Ht})^{1-!}+(1-a)("_t P_{Ft})^{1-!}}{aP_{Ht}^{1-!}+(1-a)P_{Ft}^{1-!}}\Big]^{\frac{1}{1-!}}$.

$$Q_t^{1-!} = \frac{a("_t P_{Ht})^{1-!}+(1-a)("_t P_{Ft})^{1-!}}{aP_{Ht}^{1-!}+(1-a)P_{Ft}^{1-!}}$$

Use expressions for price indices:

$$Q_t^{1-!} = \frac{a("_t \frac{1}{Z_t}\frac{W_t}{Z_t^{-1}})^{1-!}+(1-a)("_t \frac{1}{Z_t}W_t)^{1-!}}{a(\frac{1}{Z_t}\frac{W_t}{Z_t})^{1-!}+(1-a)(\frac{1}{Z_t}W_t)^{1-!}} = \Big(\frac{"_t W_t}{W_t}\Big)^{1-!}\frac{a(Z_t Z_t^{-1})^{!-1}+(1-a)Z_t^{!-1}}{aZ_t^{!-1}+(1-a)(Z_t^1 Z_t)^{!-1}} =$$

$$= \Big(\frac{\frac{"_t W_t}{P_t}P_t}{\frac{W_t}{P_t}P_t}\Big)^{1-!}\frac{a(Z_t Z_t^{-1})^{!-1}+(1-a)Z_t^{!-1}}{}$$

$$\left(\frac{a}{1\ a}\right)^{\frac{!\ +\ '\ (!\ 1)}{!\ 1}}[^{Z}$$

$$\sum_b b C_{db} \mathbf{bC} = E \, b^{DC}$$

$$+ \, E \, b \, b^b Q_t$$

$$E \quad b \, \phi_t$$

Similarly, foreign GDP y_t, i.e., output produced by home and foreign rms in the foreign country, equals $y_t = \frac{1}{1}(w_t L_t + w_t L_t)$ in units of foreign country consumption. Labor income, therefore, equals $\frac{1}{y_t}$. In units of home country consumption, this is $\frac{1}{y_t} Q_t$. The pro t of foreign rms, i.e., the pro t generated by foreign rms in home and foreign countries, $d_t + d_t$, in units of home country consumption is then $\frac{1}{y_t} Q_t$, which again shows that the share of rm pro ts, i.e., the dividend income, in the foreign GDP is a constant proportion $\frac{1}{}$.

2   Model Solution

There are four variables that will determine the model solution C_t^D, Q_t, y_t^D, and nfa_{t+1}.

2.1   Log-linearize Euler equations for consumption

Section 1.2 shows FOC wrt k_{t+1} combined with FOC wrt C_t, which gives the Euler equation:

$$C_t^{-\frac{1}{}} = E_t \{ C_{t+1}^{-\frac{1}{}} R_{t+1} \}$$

Log-linearize this equation:

$$\frac{1}{} \hat{C}_t = \frac{1}{} E_t \hat{C}_{t+1} + E_t \hat{R}_{t+1}$$

Similarly, the Euler equation for the foreign country is:

$$C_t^{-\frac{1}{}} = E_t \{ C_{t+1}^{-\frac{1}{}} R_{t+1}^f \} = E_t \{ C_{t+1}^{-\frac{1}{}} R_{t+1} \frac{Q_t}{Q_{t-1}} \}$$

where we used $R_t^f \quad \frac{v_t^f + d_t^f + d_t^f}{v_{t-1}^f}$ de ned in Section 1.4 and $R_{t+1} = \frac{Q_{t+1}}{Q_t} R_{t+1}^f$ wh17.219 7. Td [1 Tf45r1

$$E_t(\hat{C}^D_{t+1} - \hat{C}^D_t) = E_t(\hat{C}_{t+1} - \hat{C}_t)$$

2.2   Log-linearize expression from Section 1.6 and find elasticities of $\hat{C}^D_t$

This derivation finds elasticities of $\hat{C}^D_t$:

$$\frac{C_t + G_t}{C_t + G_t}\left(\frac{C_t}{C_t}\right)^{-\gamma} = \left[\frac{aZ_t^{\sigma-1} + (1-a)(Z_t^1 - Z_t)^{\sigma-1}}{a(Z_t - Z_t^{-1})^{\sigma-1} + (1-a)Z_t^{\sigma-1}}\right]^{\frac{1+\psi}{\sigma-1}}$$

$$\log(C_t + G_t) - \log(C_t + G_t) + \gamma-(\log C_t - \log C_t) = \frac{1+\psi}{\sigma-1}[\log(aZ_t^{\sigma-1} + (1-a)(Z_t^1 - Z_t)^{\sigma-1}) - \log(a(Z_t - Z_t^{-1})^{\sigma-1} + (1-a)Z_t^{\sigma-1})]$$

$$\frac{dC_t + dG_t}{C + G} - \frac{dC_t + dG_t}{C + G} + \gamma-\left(\frac{dC_t}{C} - \frac{dC_t}{C}\right) = \frac{1+\psi}{\sigma-1}[a(\sigma-1)dZ_t + (1-a)(\sigma-1)((1-\mu)dZ_t + \mu dZ_t) - [a(\sigma-1)(\mu dZ_t + (1-\mu)dZ_t) +$$

$$(1-a)(\sigma-1)dZ_t]]$$

Use $Z = Z$, which is true in the symmetric steady state. Normalize $Z = Z$ to 1.

$$\frac{dC_t\frac{C}{C} + dG_t\frac{G}{G}}{C + G} - \frac{dC_t\frac{C}{C} + dG_t\frac{G}{G}}{C + G} + \gamma-\hat{C}^D_t = \frac{1+\psi}{\sigma-1}[a(\sigma-1)\hat{Z}_t + (1-a)(\sigma-1)((1-\mu)\hat{Z}_t + \mu\hat{Z}_t) - [a(\sigma-1)(\mu\hat{Z}_t + (1-\mu)\hat{Z}_t) + (1-$$

$$a)(\sigma-1)\hat{Z}_t]$$

$$\frac{C}{C + G}(\hat{C}_t - \hat{C}_t) + \frac{G}{C + G}(\hat{G}_t - \hat{G}_t) + \gamma-\hat{C}^D_t = \frac{1+\psi}{\sigma-1}[a(\sigma-1)\hat{Z}_t + (1-a)(\sigma-1)(1-\mu)\hat{Z}_t + (1-a)(\sigma-1)\mu\hat{Z}_t - a(\sigma-1)\mu\hat{Z}_t -$$

$$a(\sigma-1)(1-\mu)\hat{Z}_t - (1-a)(\sigma-1)\hat{Z}_t]$$

Use $y = C + G$. Since $y = 1$, $C + G = 1$ and $C = 1 - G$. Then,

$$(1-G)(\hat{C}_t - \hat{C}_t) + G(\hat{G}_t - \hat{G}_t) + \gamma-\hat{C}^D_t = \frac{1+\psi}{\sigma-1}[a(\sigma-1)(1-\mu)\hat{Z}_t + (1-a)(\sigma-1)(1-\mu)\hat{Z}_t - a(\sigma-1)(1-\mu)\hat{Z}_t - a(\sigma-1)(1-\mu)\hat{Z}_t]$$

$$(1-G)\hat{C}^D_t + G\hat{G}^D_t + \gamma-\hat{C}^D_t = \frac{1+\psi}{\sigma-1}(\sigma-1)(1-\mu)(\hat{Z}_t - \hat{Z}_t)$$

$$(1-G)\hat{C}^D_t + G\hat{G}^D_t + \gamma-\hat{C}^D_t = (1+\psi)(1-\mu)\hat{Z}^D_t$$

$$(1-G+\gamma-)\hat{C}^D_t + = (1+\psi)(1-\mu)\hat{Z}^D_t - G\hat{G}^D_t$$

$$\hat{C}^D_t = \frac{(1+\psi)(1-\mu)}{1-G+\gamma-}\hat{Z}^D_t - \frac{G}{1-G+\gamma-}\hat{G}^D_t$$

$$\hat{C}^D_t = \varepsilon_{C^D Z^D}\hat{Z}^D_t + \varepsilon_{C^D G^D}\hat{G}^D_t$$

If $G = 0$ (i.e., no fiscal shocks), $\hat{C}^D_t = \frac{(1+\psi)(1-\mu)}{1+\gamma-}\hat{Z}^D_t$

If $G = 0$ and $\psi = 0$ (i.e., inelastic labor) $\hat{C}^D_t = (1-\mu)\hat{Z}^D_t$

If $G = 0$, $\psi = 0$, and $\mu = 1$, $\hat{C}^D_t = 0$.

If $G = 0$, $\psi = 0$, and $\mu = 0$, $\hat{C}^D_t = \hat{Z}^D_t$.

If $G = 0$ and $\mu = 1$, $\hat{C}^D_t = 0$ regardless of ψ.

If $G \neq 0$ and $\psi = 0$, $\hat{C}^D_t = \frac{(1-\mu)}{1-G}\hat{Z}^D_t - \frac{G}{1-G}\hat{G}^D_t$

If $G \neq 0$, $\psi = 0$ and $\mu = 1$, $\hat{C}^D_t = -\frac{G}{1-G}\hat{G}^D_t$. If $\mu = 0$, $\hat{C}^D_t = \frac{1}{1-G}\hat{Z}^D_t - \frac{G}{1-G}\hat{G}^D_t$.

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2.3 Find elasticities of $\hat{Q}_t$

This derivation uses the log-linearized Euler equations from Section 2.1 and $\hat{C}_t^D$ from Section 2.2 to find elasticities of $\hat{Q}_t$:

$$E_t(\hat{C}_{t+1}^D - \hat{C}_t^D) = E_t(\hat{Q}_{t+1} - \hat{Q}_t) \text{ from Section 2.1.}$$

Combine with $\hat{C}_t^D = c_D z_D \hat{Z}_t^D + c_D g_D \hat{G}_t^D$ from 2.2.

$$E_t(\hat{Q}_{t+1} - \hat{Q}_t) = E_t[\, c_D z_D (\hat{Z}_{t+1}^D - \hat{Z}_t^D) + c_D g_D \hat{G}_{t+1} - \hat{G}_t^D$$

Log-linearized: $b_t^{total;D} = b_t^D - b_t^{*D}$.

2.6 Log-linearize NFA LOM

This derivation uses the NFA LOM from Section 1.4 to find the solution form nfa_{t+1}:

$nfa_{t+1} = R_t^D + R_t nfa_t + (1 - a)[(y_t - Q_t y_t^f) - (C_t - Q_t C_t^f) - (G_t - Q_t G_t^f)]$

$dnfa_{t+1} = dR_t^D + R^D d_t + dR_t nfa + R dnfa_t + (1 - a)[dy_t - (dQ_t y^f + Q dy_t^f) - (dC_t$
$- (dQ_t C^f + Q dC_t^f)) - (dG_t - (dQ_t G^f + Q dG_t^f)]$

Use $R^D = 0$ and $nfa = 0$:

$dnfa_{t+1} = dR_t^D + R dnfa_t + (1 - a)[dy_t - (dQ_t y^f + Q dy_t^f) - (dC_t - (dQ_t G^f + Q dG_t^f))]$
$- (dG_t - (dQ_t G^f + Q dG_t^f)]$

Use $Q = 1$ because it holds in the symmetric steady state, and net foreign assets equal 0:

$dnfa_{t+1} = dR_t^D + R dnfa_t + (1 - a)[(dy_t - (dQ_t y^f + dy_t^f) - (dC_t - (dQ_t C^f + dC_t^f))$
$- (dG_t - (dQ_t G^f + dG_t^f)]$

Notice that we are subtracting dy_t and dy_t^f that are in units of home consumption and foreign consumption, respectively. We can subtract these terms because we already accounted for the different units by including the real exchange rate. This works because in the symmetric steady state, the real exchange rate terms drop out ($Q = 1$). This is used later on in other derivations, for example, the derivation of the differential in equity values b_t^D.

$dnfa_{t+1} = dR_t^D + R dnfa_t + (1 - a)[(dy_t^D - dQ_t y^f) - (dC_t^D - dQ_t C^f) - (dG_t^D - dQ_t G^f)]$

Divide by C. Use $C = 1 - G$, which comes from $y = C + G$ combined with $y = 1$:

$\frac{dnfa_{t+1}}{C} = \frac{dR_t^D}{1 - G} + \frac{R dnfa_t}{C} + (1 - a)[(\frac{dy_t^D}{1 - G} - \frac{dQ_t y^f}{1 - G}) - (\frac{dC_t^D}{C} - \frac{dQ_t C^f}{C}) - (\frac{dG_t^D}{1 - G} - \frac{dQ_t G^f}{1 - G})]$

$nfa_{t+1} = \frac{dR_t^D}{1 - G} R$

$$nfa_{t+1} = \frac{b}{(1-G)} R_t^D + {}^1 nfa_t + \frac{1-a}{1-G} y_t^D \quad (1-a)c_t^D \quad \frac{(1-a)G}{1-G} c_t^D + (1-a)\left[\quad \frac{q_t}{1-G} + \frac{q_t(1-G^f)}{1-G} + \frac{q_t G^f}{1-G} \right]$$

$$nfa_{t+1} = \frac{b}{(1-G)} R_t^D$$

$$\frac{dv_t}{v} = \frac{dE_t v_{t+1}}{v} + \frac{dE_t d_{t+1}}{v} + \frac{dE_t d_{t+1}}{v}$$

$$v_t = E_t v_{t+1} + E_t d_{t+1}\frac{d}{v} + E_t d_{t+1}\frac{d}{v}$$

From Section 1.7, the following holds: $d_t + d_t = {}^1 y_t$. Due to the assumption $y_t = 1$, it is possible to write: $d_t + d_t = {}^1$. In steady state, the Euler equation for home shares becomes $v = v + d + d$, which becomes $v = v + {}^1$ which can be written as $v(1 \quad) = -$
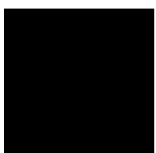
Next, we obtain an expression for $\hat{d}^D_{t+1}$. Here, we take advantage of the useful properties from Section 1.7. Since $\bar{d}_t = d_t + d_t = {}^1 y_t$ and $\bar{d}_t = d_t + d_t = {}^1 y_t Q_t$ in units of home country consumption, it is possible to write $\frac{\bar{d}_t}{d_t} = \frac{d_t + d_t}{d_t + d_t} = \frac{{}^1 y_t}{{}^1 y_t Q_t}$, which means $\frac{\bar{d}_t}{d_t} = \frac{y_t}{y_t Q_t}$. Roll it forward by one period: $\frac{\bar{d}_{t+1}}{d_{t+1}} = \frac{y_{t+1}}{y_{t+1} Q_{t+1}}$.

Log-linearizing gives $\hat{d}^D_{t+1} = \hat{y}_{t+1} \quad (\hat{y}_{t+1} + \hat{Q}_{t+1})$.

Substitute into $\hat{u}^D_t$:

$\hat{u}^D_t = E_t[\ \hat{u}^D_{t+1} + (1 \quad)(\hat{y}_{t+1} \quad (\hat{y}_{t+1} + \hat{Q}_{t+1}))]$

Notice: This combines $E_t \hat{R}^D_{t+1} = 0$ and $\hat{R}^D_t = \ [\ \hat{u}^D_t + (1 \quad)(\hat{y}_t \quad (\hat{y}_t + \hat{Q}_t))] + \hat{u}^D_{t\ 1} =$

$= \ [\ \hat{u}^D_t + (1 \quad)(\hat{y}^D_t \quad \hat{Q}_t)] + \hat{u}^D_{t\ 1}$

$\hat{u}^D_t = E_t[\ \hat{u}^D_{t+1} + (1 \quad)(\hat{y}^D_{t+1} \quad \hat{Q}_{t+1})]$

$\hat{u}^D_t = E_t[\ \hat{u}^D_{t+1} + (1 \quad)(\ _{y^D Z^D} \hat{Z}^D_{t+1} + \ _{y^D G^D} \hat{G}^D_{t+1} \quad _{QZ^D} \hat{Z}^D_{t+1} \quad _{QG^D} \hat{G}^D_{t+1}]$

$\hat{u}^D_t = E_t[\ \hat{u}^D_{t+1} + (1 \quad)(\ _{y^D Z^D} \hat{Z}^D_{t+1} + \ _{y^D G^D} \hat{G}^D_{t+1} \quad ^1 \ _{C^D Z^D} \hat{Z}^D_{t+1} \quad ^1 \ _{C^D G^D} \hat{G}^D_{t+1}]$

$\hat{u}^D_t = E_t[\ \hat{u}^D_{t+1} + (1 \quad)((\ _{y^D Z^D} \quad ^1 \ _{C^D Z^D})\hat{Z}^D_{t+1} + (\ _{y^D G^D} \quad ^1 \ _{C^D G^D})\hat{G}^D_{t+1}]$

$\hat{u}^D_t = \ _{v^D Z^D} \hat{Z}^D_t + \ _{v^D G^D} \hat{G}^D_t$

$_{v^D Z^D} \hat{Z}^D_t + \ _{v^D G^D} \hat{G}^D_t = E_t[\ \hat{u}^D_{t+1} + (1 \quad)((\ _{y^D Z^D} \quad ^1 \ _{C^D Z^D})\hat{Z}^D_{t+1} + (\ _{y^D G^D} \quad ^1 \ _{C^D G^D})\hat{G}^D_{t+1}]$

$_{v^D Z^D} \hat{Z}^D_t + \ _{v^D G^D} \hat{G}^D_t = \ (\ _{v^D Z^D} \ _z \hat{Z}^D_t + \ _{v^D G^D} \ _G \hat{G}^D_t) + (1 \quad)[(\ _{y^D Z^D} \quad ^1 \ _{C^D Z^D}) \ _z \hat{Z}^D_t +$
$(\ _{y^D G^D} \quad ^1 \ _{C^D G^D}) \ _G \hat{G}^D_t]$

where we used $\hat{Z}^D_{t+1} = \ _z \hat{Z}^D_t + \ _{Z^D\ t+1}$ and $\hat{G}^D_{t+1} = \ _G \hat{G}^D_t + \ _{G^D\ t+1}$

Match the coefficients:

$_{v^D Z^D} = \ _{v^D Z^D} \ _z + (1 \quad)(\ _{y^D Z^D} \quad ^1 \ _{C^D Z^D}) \ _z$

$(1 \quad _z) \ _{v^D Z^D} = (1 \quad)(\ _{y^D Z^D} \quad ^1 \ _{C^D Z^D}) \ _z$

$_{v^D Z^D} = \dfrac{(1 \quad) \ _z (\ _{y^D Z^D} \quad ^1 \ _{C^D Z^D})}{1 \quad _z}$

$_{v^D G^D} = \ _{v^D G^D} \ _G + (1 \quad)(\ _{y^D G^D} \quad ^1 \ _{C^D G^D}) \ _G$

$(1 \quad _G) \ _{v^D G^D} = (1 \quad)(\ _{y^D G^D} \quad ^1 \ _{C^D G^D}) \ _G$

$_{v^D G^D} = \dfrac{(1 \quad) \ _G (\ _{y^D G^D} \quad ^1 \ _{C^D G^D})}{1 \quad _G}$

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2.8 Show that excess return R_t^D is a linear function of innovations to relative productivity and government spending

From Section 2.7:

$$R_{t+1}^D = [\phi_{t+1}^D + (1 \quad)(\psi_{t+1}^D \quad b$$

If $G \neq 0$, $' = 0$ and $= 1$: $\hat{R}^D_{t+1} = \frac{(1\ )(1\ )G}{(1\ z)(1\ G)}\hat{b}_{ZD\,t+1}\quad \frac{(1\ )G}{(1\ G)(1\ G)}\hat{b}_{GD\,t+1}$

2.9 2nd-order approximation of the portfolio part of the model

From household FOCs for :

$C_t^{\ 1} = E_t f C_{t+1}^{\ 1} R_{t+1} g$, which can be written as: $\frac{C_t^{\ 1}}{} = E_t f C_{t+1}^{\ 1} R_{t+1} g$

$C_t^{\ 1} = E_t f C_{t+1}^{\ 1} R_{t+1} g$, which can be written as: $\frac{C_t^{\ 1}}{} = E_t f C_{t+1}^{\ 1} R_{t+1} g$

Equating these two expressions gives us $E_t(C_{t+1}^{\ 1} R_{t+1}) = E_t(C_{t+1}^{\ 1} R_{t+1})$, which can be written as $E_t(C_{t+1}^{\ 1} R_{t+1})\quad E_t(C_{t+1}^{\ 1} R_{t+1}) = 0$

Take second-order approximation and evaluate it at steady state:

$E_t(\ ^1 C_{t+1}^{\ 1}\ ^1 dC_{t+1} R_{t+1}) + E_t(C_{t+1}^{\ 1} dR_{t+1})\quad E_t(\ ^1 C_{t+1}^{\ 1}\ ^1 dC_{t+1} R_{t+1})\quad E_t(C_{t+1}^{\ 1} dR_{t+1}) +$

$+ \frac{1}{2}[\ ^1(\ ^1\ 1)C_{t+1}^{\ 1}\ ^1\ ^1 d^2 C_{t+1} R_{t+1} + C_{t+1}^{\ 1} 0 + 2(\ ^1)C_{t+1}^{\ 1}\ ^1 dC_{t+1} dR_{t+1}]$

$\frac{1}{2}[\ ^1(\ ^1\ 1)C_{t+1}^{\ 1}\ ^1\ ^1 d^2 C_{t+1} R_{t+1}\quad C_{t+1}^{\ 1} 0\quad 2(\ ^1)C_{t+1}^{\ 1}\ ^1 dC_{t+1} dR_{t+1}] =$

$= E_t(\ ^1 C^{\ 1}\ ^1 dC_{t+1}\ ^1) + E_t(C^{\ 1} dR_{t+1})\quad E_t(\ ^1 C^{\ 1}\ ^1 dC_{t+1}\ ^1)\quad E_t(C^{\ 1} dR_{t+1}) +$

$+ \frac{1}{2}[\ ^1(\ ^1\ 1)C^{\ 1}\ ^1\ ^1 d^2 C_{t+1}\ ^1 + 2(\ ^1)C^{\ 1}\ ^1 dC_{t+1} dR_{t+1}]$

$\frac{1}{2}[\ ^1(\ ^1\ 1)C^{\ 1}\ ^1\ ^1 d^2 C_{t+1}\ ^1 + 2(\ ^1)C^{\ 1}\ ^1 dC_{t+1} dR_{t+1}] =$

$= E_t(C^{\ 1} dR_{t+1})\quad E_t(C^{\ 1} dR_{t+1}) + \frac{1}{2}[2(\ ^1)C^{\ 1}\ ^1 dC_{t+1} dR_{t+1}]\quad \frac{1}{2}[2(\ ^1)C^{\ 1}\ ^1 dC_{t+1} dR_{t+1}]$

$= E_t(C^{\ 1} dR_{t+1})\quad E_t(C^{\ 1} dR_{t+1}) + [\ ^1 C^{\ 1}\ ^1 dC_{t+1} dR_{t+1}]\quad [\ ^1 C^{\ 1}\ ^1 dC_{t+1} dR_{t+1}]$

Divide by $C^{\ 1}$ and R.

$\hat{R}_{t+1}\quad \hat{R}_{t+1} + (\ ^1\hat{C}_{t+1}\ \hat{R}_{t+1})\quad (\ ^1\hat{C}_{t+1}\ \hat{R}_{t+1}) = 0$

The same derivation for the foreign country gives:

$\hat{R}^f_{t+1}\quad \hat{R}^f_{t+1} + (\ ^1\hat{C}^f_{t+1}\ \hat{R}^f_{t+1})\quad (\ ^1\hat{C}^f_{t+1}\ \hat{R}^f_{t+1}) = 0$

Subtract expressions for the home and foreign countries:

$\hat{R}_{t+1}\quad \hat{R}_{t+1} + (\ ^1\hat{C}_{t+1}\ \hat{R}_{t+1})\quad (\ ^1\hat{C}_{t+1}\ \hat{R}_{t+1})\quad [\hat{R}^f_{t+1}\quad \hat{R}^f_{t+1} + (\ ^1\hat{C}^f_{t+1}\ \hat{R}^f_{t+1})\quad (\ ^1\hat{C}^f_{t+1}\ \hat{R}^f_{t+1})] =$

0

From Section 1.4: $R_{t+1} = \frac{Q_{t+1}}{Q_t} R^f_{t+1}$

Log-linearize: $\hat{R}_{t+1} = \hat{Q}_{t+1}\quad \hat{Q}_t + \hat{R}^f_{t+1}$

Same for the foreign:

Log-linearize: $\hat{R}_{t+1} = \hat{Q}_{t+1} \quad \hat{Q}_t + \hat{R}_{t+1}^f$

Using this, simplify: $(\quad^1\hat{Q}_{t+1} \hat{R}_{t+1}) \quad (\quad^1\hat{Q}_{t+1} \hat{R}_{t+1}) \quad [\quad^1\hat{Q}_{t+1}^f \hat{R}_{t+1}^f) \quad (\quad^1\hat{Q}_{t+1}^f \hat{R}_{t+1}^f)] = 0$

$\hat{Q}_{t+1} \hat{R}_{t+1} \quad \hat{Q}_{t+1} \hat{R}_{t+1} \quad [\hat{Q}_{t+1}^f \hat{R}_{t+1}^f \quad \hat{Q}_{t+1}^f \hat{R}_{t+1}^f] = 0$

$\hat{Q}_{t+1} \hat{R}_{t+1} \quad \hat{Q}_{t+1} \hat{R}_{t+1} \quad [\hat{Q}_{t+1}^f (\hat{R}_{t+1} \quad \hat{Q}_{t+1} + \hat{Q}_t) \quad \hat{Q}_{t+1}^f (\hat{R}_{t+1} \quad \hat{Q}_{t+1} + \hat{Q}_t)] = 0$

$\hat{Q}_{t+1} \hat{R}_{t+1} \quad \hat{Q}_{t+1} \hat{R}_{t+1} \quad [\hat{Q}_{t+1}^f \hat{R}_{t+1} \quad \hat{Q}_{t+1}^f \hat{R}_{t+1}] = 0$

$E_t(\hat{Q}_{t+1}^D \hat{R}_{t+1}^D) = 0$

This results is the same as in GLR.

However, notice that there is no in either expression:

Substitute expressions for $\hat{Q}_{t+1}^D$ from Section 2.2 (i.e., $\hat{Q}_{t+1}^D = \frac{(1+')(1)}{1\ G+} \hat{Z}_{t+1}^D \quad \frac{G}{1\ G+} \hat{Q}_{t+1}^D$)

and $\hat{R}_{t+1}^D$ from Section 2.8 (i.e., $\hat{R}_{t+1}^D = \frac{(1\ )\frac{(1+')(1\ )[\ (1\ G)\ 1]}{(1\ G+\ -)}}{1\ z} \hat{b}_{Z^D\ t+1} \quad \frac{(1\ )\frac{G('\ +1)}{+(P\ t\ 6\ 4\ -)}}{1\ G}$

Hence,

$$\hat{y}_t = a\left[\widehat{RP}_t + \hat{Z}_t + \hat{L}_t\right]^h + (1-a)\left[\widehat{RP}_t + (1-)\hat{Z}_t + \hat{Z}_t + \hat{L}_t\right]^i; \qquad (4)$$

$$\hat{y}_t = a\left[\widehat{RP}_t + \hat{Z}_t + (1-)\hat{Z}_t + \hat{L}_t\right]^h + (1-a)\left[\widehat{RP}_t + \hat{Z}_t + \hat{L}_t\right]^i: \qquad (5)$$

Next, take a population-weighted average of equations (4) and (5), and de ne $\hat{y}_t^W$ as:

$$\hat{y}_t^W \quad a\hat{y}_t + (1-a)\hat{y}_t =$$

n)Tf 18.18 -1.793 Td [( )]TJ/F24 11.9552 Tf 11.955 0 Td [( )]TJ/F18 11.9552 Tf 6.722 0 Td

= a

that this is the same system of equations as in GLR. It follows that the change in production structure and demand-ful llment from the one in GLR to the one we are studying in this paper matters for how world production is allocated between the two countries but not for the overall amount of world production.

References

Ghironi, F., Lee, J., & Rebucci, A. (2015). The valuation channel of external adjustnfences